The Chaos Game

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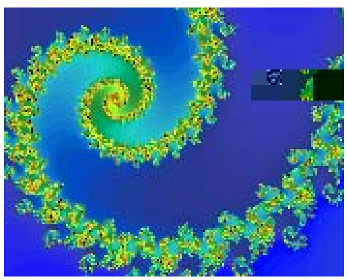


Grade Level	6-8, 9-12
Subject Area	math
Curriculum Focus	computer science, physical science, life science
Duration	2 hours

Objective

Students will take measurements and make predictions to describe the behavior of a fractal known as the Sierpinski triangle (S); develop a deterministic construction of S; become familiar with a self-similar object (a fractal) by playing �The Chaos Game� and determine an algorithm for the fractal.

Materials

dice, with the sides of each die colored so that two sides are red, two blue, and two green (it doesn�t matter which two sides); colored markers; pencil and paper; computer with Internet access;student worksheets

Background

When we look at a structure such as a bridge or a skyscraper, we may notice how strong, durable, and motionless it seems. What we don�t notice are the vibrations in the structure. Harmless under normal conditions, these vibrations can stretch and contract the structure. Engineers plan for this on large construction projects, but sometimes nature can send this normally graceful sway out of control. A dramatic example of this happened on November 7, 1940. Around 11 a.m., the first Tacoma Narrows suspension bridge�situated on the Tacoma Narrows in Puget Sound in Washington�collapsed. The normal vibrations of the bridge were sent into a wild resonance by winds of 42 miles per hour, causing the structure to act like a giant slinky. Eventually the motion was too much for the structure and it collapsed violently into the sound. This disaster reminds us that well-defined patterns can sometimes become random, chaotic events�often beyond human control.

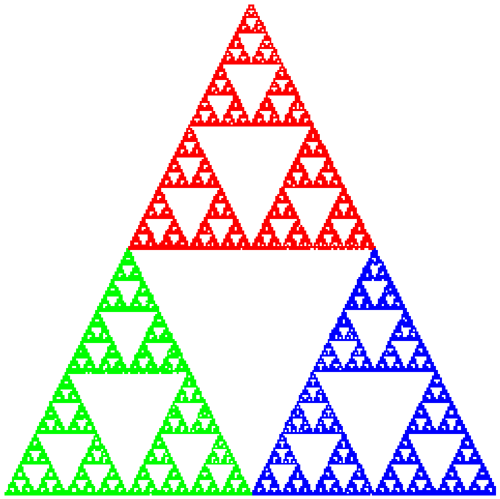
But what about random events? Can they lead to well-defined patterns? This activity will provide an opportunity to see some chaos in action and to explore a surprising underlying structure known as the Sierpinski triangle using a Java-based game.

But first, it�s good to keep these three things in mind:

- *Chaos theory*has become a popular phrase among scientists and mathematicians. One of the goals of research in chaos theory is to understand patterns in nature.
- A*fractal*is a very irregular curve or shape that has self-similarity�that is, any suitably chosen part is similar in shape to a given larger or smaller part when magnified or reduced to the same size. This term was first coined by the French mathematician Benoit Mandelbrot, who has a fractal named after him.
- An*algorithm*is a step-by-step procedure for solving a problem in a finite number of steps.

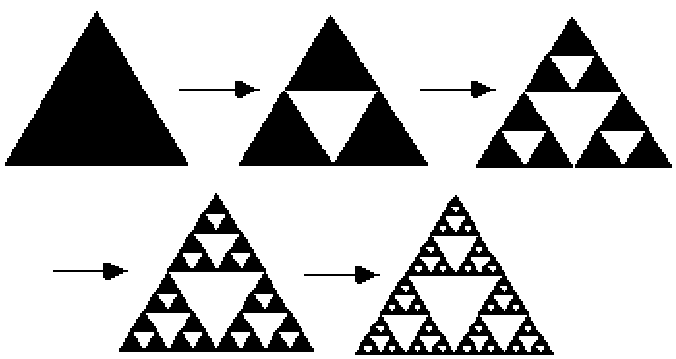
Procedure

1. Divide the class into teams of three or four. Give each team a single die, three colored markers, a ruler, paper, and part one of thestudent worksheets. Instruct teams to have one student read the rules of the game and begin.
2. Teams will start by drawing a triangle, coloring the vertices, selecting one vertex as a starting point, and rolling the colored die to determine where the next point will be. After about five or six moves, the teams will start tracking the movement of the points after each die roll�they�ll need to track points for 50 rolls. When this is completed, teams should compare their results (orbits) and make their predictions of what the resulting shape would be if they were to individually track 100 or 1,000 points or combine orbits (seeworksheet answer key).
3. Display the Sierpinski triangle to the class:



Explain that after many hundreds or thousands of points are plotted, the result is this well-defined shape�which is anything but random. The spooky thing is that it works*every time*! No matter where you start (even outside the triangle), if you keep rolling the die enough times, eventually you will get this shape. You can demonstrate this to the class by showing thisdemo.

4. Have the teams work together to come up with their own simple set of rules that will achieve the same shape without relying on random points. This is known as a*deterministic construction.*When finished, have the teams share their rules. One method is to start with a triangle and select the midpoints of each side to form a new triangle inside the larger one. The new triangle is removed and three triangles remain (which are all similar to the larger triangle). The process is repeated for the remaining three triangles, which yield nine triangles when their middle ones are removed, and so on, as small as you can go:



5. Now, invite student teams to become familiar with the self-similar nature of fractals by playing a target-shooting challenge known asThe Chaos Game.They should first read the instructions for the game inpart 2of the student handout. As students play the game, help them see that each time they move the dot and plot a new point, they are in essence shrinking the figure into a smaller triangle

with the same proportions as the larger triangle. This �self-similarity� prevails no matter how many points are plotted or how many different smaller triangles are created within the starting triangle.

6. After playing the Chaos Game a few times, have students return to their teams. The teams should discuss and write up a strategy for hitting the target in the fewest number of moves at the novice level, writing out an algorithm to express their strategy. Select volunteers to present their strategies to the class.

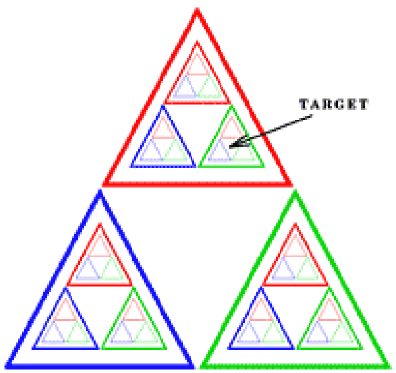
Extension

Fractals abound in nature. Invite student teams to find examples of fractals in the physical world and share them (or pictures of them) with the class. Remind them that the shape of the object or curve must be irregular and self-similar. Some possibilities are sea shells, shorelines, clouds, mountains, leaves, trees, and crystals.

Worksheet Answer Key

Part 1: Students may guess that the resulting shape will be completely random or that it will fill the space entirely, especially when combined with other teams. But the result is surprising. Over the course of hundreds of iterations (rolls of the die), the shape of the resulting figure becomes clear. It�s a figure known as the Sierpinski triangle! What appeared to be random actually had an invisible pattern to it.

Part 2:
The fewest number of moves needed to hit the target on the novice level is four. Although different strategies may work, one possible algorithm involves coloring all the triangles on the board.



The Java program�s algorithm works by finding the midpoints of all the vertices and then calling a triangle routine on the three triangles created by the midpoints. In the picture above, these three triangles are the bigger red, blue, and green triangles. All of these triangles are then divided into three smaller triangles: one red, one green, one blue. This process is repeated until the depth reaches a predefined limit.

In this example, the target in the picture is in a red triangle, and then a green triangle within that, and then a blue triangle. Mapping the dots just requires that we reverse the order: blue, green, red.

Related Links

Chaos in the Classroom
http://math.bu.edu/DYSYS/chaos-game/chaos-game.html

Art Matrix Fractals
http://www.artmatrix.com/

Fractal Images on the Internet http://www.ba.infn.it/www/wfractal.html

Credits

�The Chaos Game� was conceived by Kevin Lee of theCollege of St. Catherinein St. Paul, Minnesota. The Java applets appear courtesy of Johanna Voolich, Adrian Vajiac, and Robert L. Devaney of Boston University�sDepartment of Mathematics.